Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME									
CENTER NUMBER						CANDIDATE NUMBER				

9202252369

MATHEMATICS (US) **0444/41**

Paper 4 (Extended) **May/June 2016**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments
Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Center number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If work is needed for any question it must be shown in the space provided.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant digits.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

The number of points is given in parentheses [ ] at the end of each question or part question.
The total of the points for this paper is 130.

Write your calculator model in the box below.

This document consists of **19** printed pages and **1** blank page.

CAMBRIDGE International Examinations

Formula List

For the equation $ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Lateral surface area, A, of cylinder of radius r, height h. $\quad A = 2\pi rh$

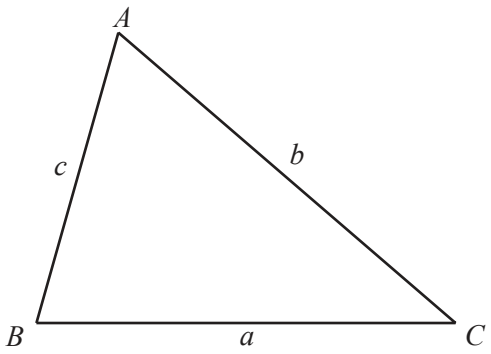
Lateral surface area, A, of cone of radius r, sloping edge l. $\quad A = \pi rl$

Surface area, A, of sphere of radius r. $\quad A = 4\pi r^2$

Volume, V, of pyramid, base area A, height h. $\quad V = \frac{1}{3}Ah$

Volume, V, of cone of radius r, height h. $\quad V = \frac{1}{3}\pi r^2 h$

Volume, V, of sphere of radius r. $\quad V = \frac{4}{3}\pi r^3$

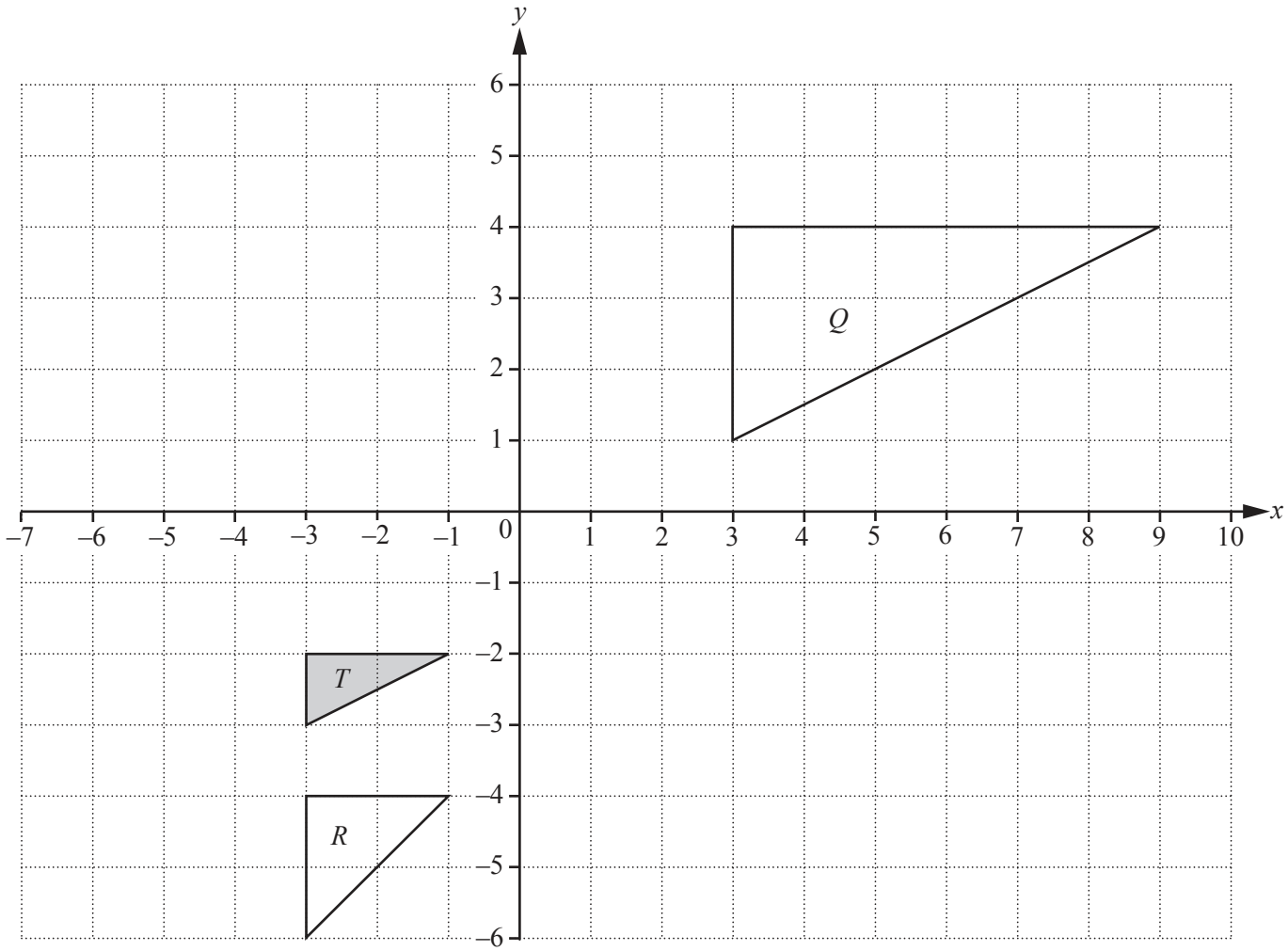


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area} = \frac{1}{2}bc\sin A$$

1



(a) Draw the image of triangle T after a translation by the vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$. [2]

(b) Draw the image of triangle T after a reflection in the line $y = 1$. [2]

(c) Describe fully the **single** transformation that maps triangle T onto triangle Q.

.....

..... [3]

(d) Describe fully the **single** transformation that maps triangle T onto triangle R.

.....

..... [3]

2 **(a)** Kristian and Stephanie share some money in the ratio 3 : 2.
Kristian receives $72.

(i) Work out how much Stephanie receives.

$ ..... [2]

(ii) Kristian spends 45% of his $72 on a computer game.

Calculate the price of the computer game.

$ ..... [1]

(iii) Kristian also buys a meal for $8.40 .

Calculate the fraction of the $72 Kristian has left after buying the computer game and the meal.
Give your answer in its lowest terms.

..... [2]

(iv) Stephanie buys a book in a sale for $19.20 .
This sale price is after a reduction of 20%.

Calculate the original price of the book.

$ ..... [3]

(b) Boris invests $550 at a rate of 2% per year simple interest.

Calculate the amount Boris has after 10 years.

$ ..... [3]

(c) Marlene invests $550 at a rate of 1.9% per year compound interest.

Calculate the amount Marlene has after 10 years.

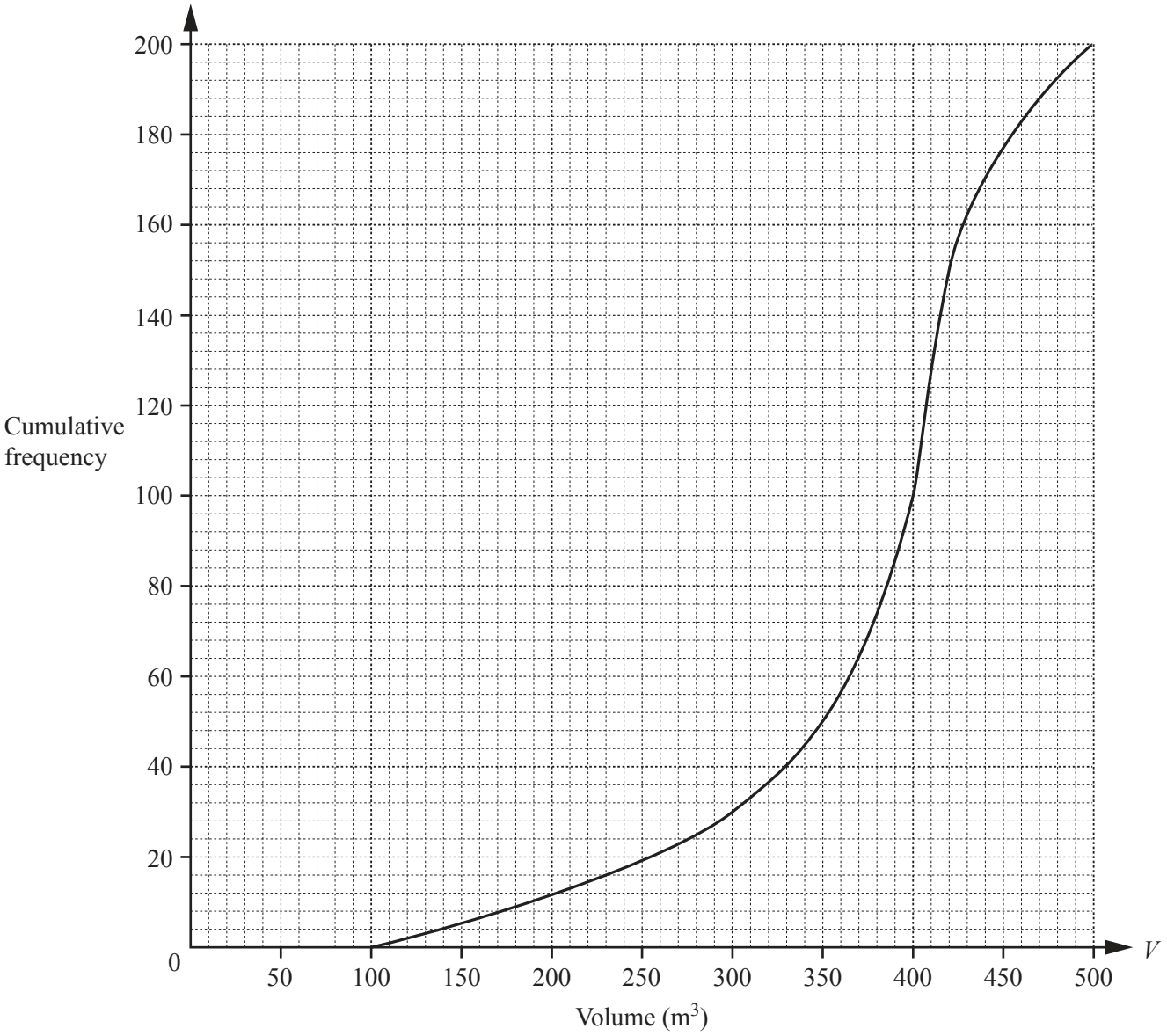
$ ..... [2]

(d) Hans invests $550 at a rate of x% per year compound interest.
At the end of 10 years he has a total amount of $638.30, correct to the nearest cent.

Find the value of x.

$x =$ ..... [3]

3 **(a)** 200 students estimate the volume, V m^3, of a classroom.
The cumulative frequency diagram shows their results.



Find

(i) the median,

..... m^3 [1]

(ii) the lower quartile,

..... m^3 [1]

(iii) the inter-quartile range,

..... m^3 [1]

(iv) the number of students who estimate that the volume is greater than 300 m^3.

..... [2]

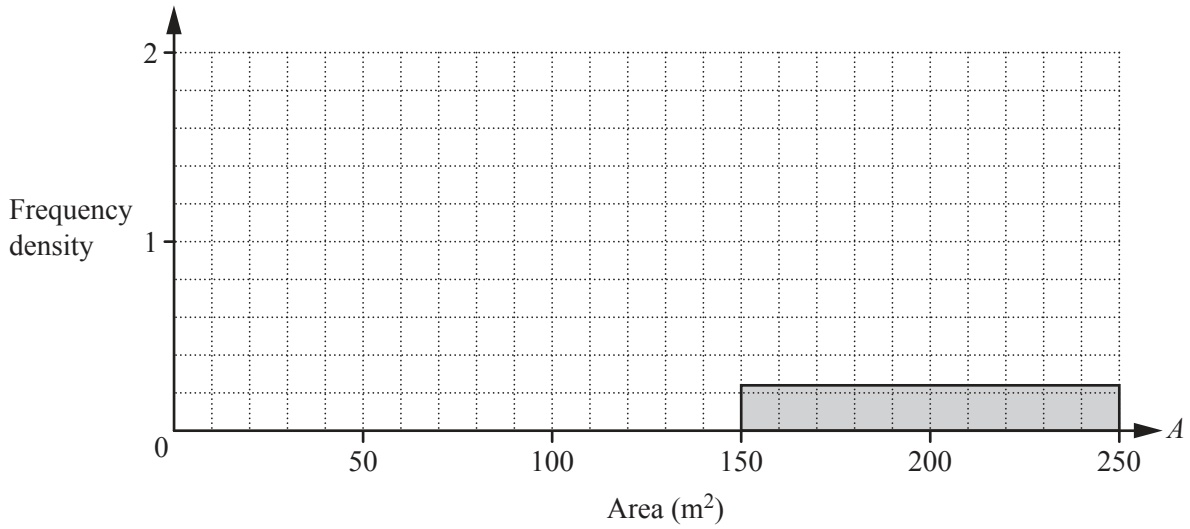
(b) The 200 students also estimate the total area, $A\,m^2$, of the windows in the classroom.
The results are shown in the table.

Area ($A\,m^2$)	$20 < A \leqslant 60$	$60 < A \leqslant 100$	$100 < A \leqslant 150$	$150 < A \leqslant 250$
Frequency	32	64	80	24

(i) Calculate an estimate of the mean.
Show all your working.

..... m^2 [4]

(ii) Complete the histogram to show the information in the table.



[4]

(iii) Two of the 200 students are chosen at random.

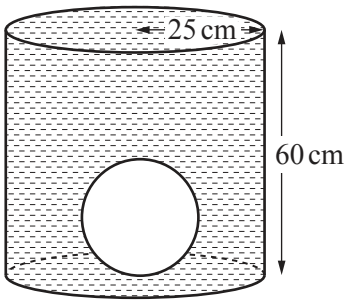
Find the probability that they both estimate that the area is greater than $100\,m^2$.

..... [2]

4 **(a)** Calculate the volume of a metal sphere of radius 15 cm and show that it rounds to 14 140 cm^3, correct to 4 significant figures.

[2]

(b) **(i)** The sphere is placed inside an empty cylindrical tank of radius 25 cm and height 60 cm. The tank is filled with water.

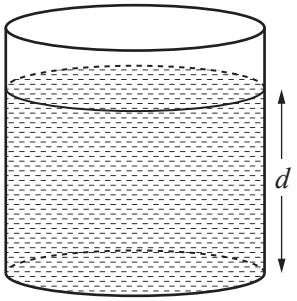


NOT TO SCALE

Calculate the volume of water required to fill the tank.

..... cm^3 [3]

(ii) The sphere is removed from the tank.



NOT TO SCALE

Calculate the depth, d, of water in the tank.

d = ... cm [2]

(c) The sphere is melted down and the metal is made into a solid cone of height 54 cm.

(i) Calculate the radius of the cone.

..... cm [3]

(ii) Calculate the **total** surface area of the cone.

..... cm^2 [4]

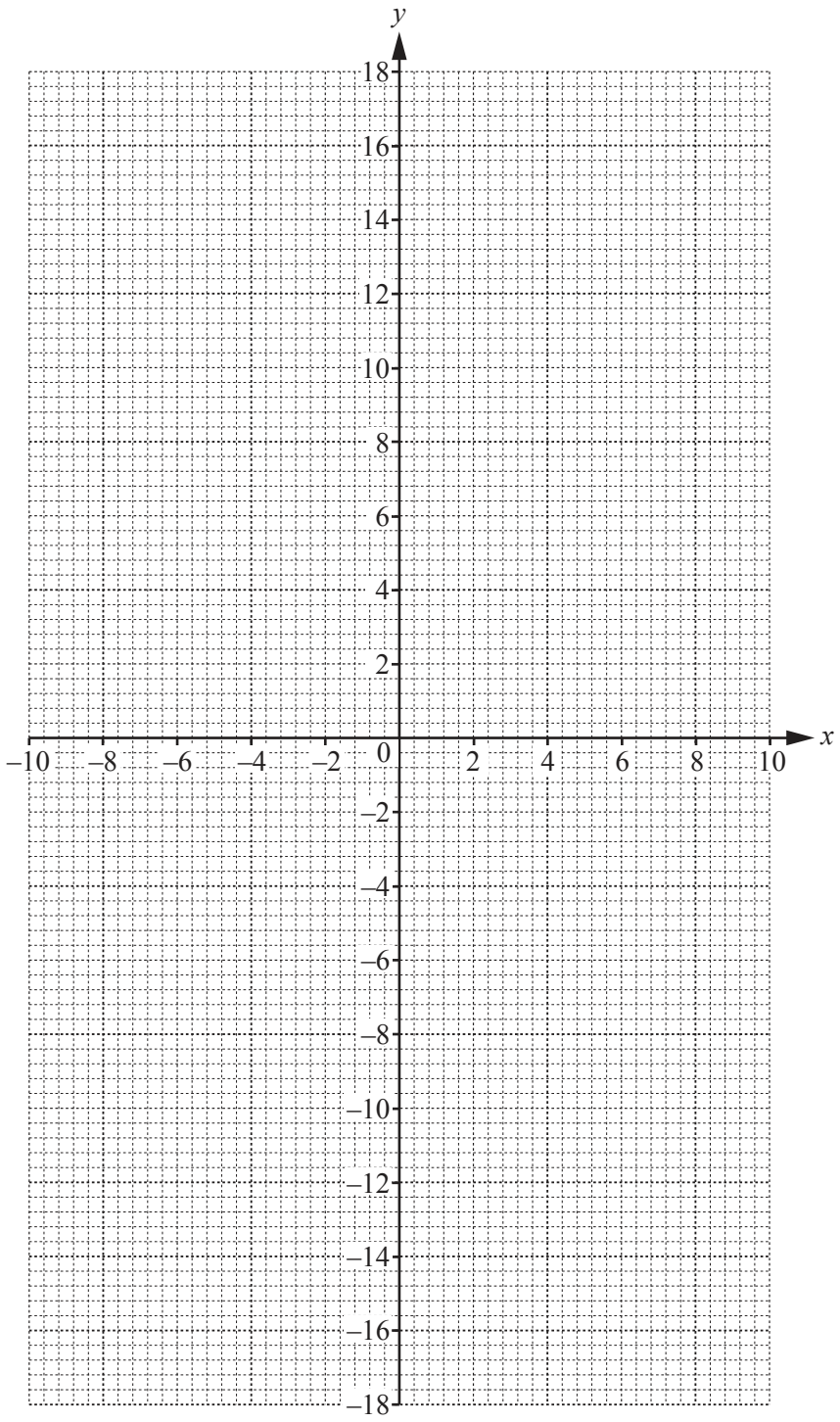
5 $f(x) = \frac{20}{x} + x, \quad x \neq 0$

(a) Complete the table.

x	-10	-8	-5	-2	-1.6		1.6	2	5	8	10
$f(x)$	-12	-10.5	-9	-12	-14.1		14.1	12			12

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leqslant x \leqslant -1.6$ and $1.6 \leqslant x \leqslant 10$.



[5]

(c) Using your graph, solve the equation $f(x) = 11$.

$x =$ ..... or $x =$ ..... [2]

(d) k is a prime number and $f(x) = k$ has no solutions.

Find the possible values of k.

..... [2]

(e) The slope of the graph of $y = f(x)$ at the point (2, 12) is -4.

Write down the co-ordinates of the other point on the graph of $y = f(x)$ where the slope is -4.

(..... , ...) [1]

(f) **(i)** The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$.

Show that $p = -1$ and $q = -20$.

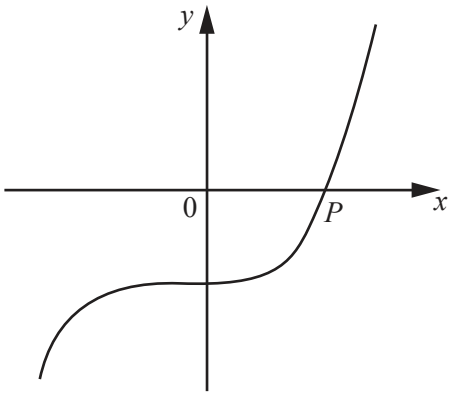
[2]

(ii) On the grid opposite, draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$. [2]

(iii) Using your graphs, solve the equation $x^3 - x^2 - 20 = 0$.

$x =$ ..... [1]

(iv)



NOT TO SCALE

The diagram shows a **sketch** of the graph of $y = x^3 - x^2 - 20$.
P is the point $(n, 0)$.

Write down the value of n.

$n =$ ..... [1]

6 (a)

NOT TO SCALE

The perimeter of the rectangle is 80 cm.
The area of the rectangle is A cm^2.

(i) Show that $x^2 - 40x + A = 0$.

[3]

(ii) When $A = 300$, solve, by factoring, the equation $x^2 - 40x + A = 0$.

$x =$ ..... or $x =$ .....[3]

(iii) When $A = 200$, solve, by using the quadratic formula, the equation $x^2 - 40x + A = 0$.
Show all your working and give your answers correct to 2 decimal places.

$x =$ ..... or $x =$ .....[4]

(b) A car completes a 200 km journey with an average speed of x km/h.
The car completes the return journey of 200 km with an average speed of $(x + 10)$ km/h.

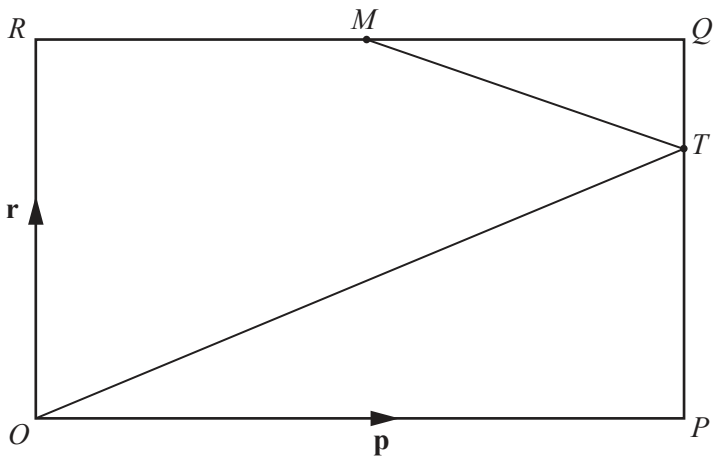
(i) Show that the difference between the time taken for each of the two journeys is $\frac{2000}{x(x+10)}$ hours.

[3]

(ii) Find the difference between the time taken for each of the two journeys when $x = 80$.
Give your answer in **minutes** and **seconds**.

..... min ..... s [3]

7



NOT TO SCALE

$OPQR$ is a rectangle and O is the origin.
M is the midpoint of RQ and $PT : TQ = 2 : 1$.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of **p** and/or **r**, in its simplest form

(i) $\overrightarrow{MQ}$,

$\overrightarrow{MQ} =$ ..... [1]

(ii) $\overrightarrow{MT}$,

$\overrightarrow{MT} =$ ..... [1]

(iii) $\overrightarrow{OT}$.

$\overrightarrow{OT} =$ ..... [1]

(b) RQ and OT are extended to meet at U.

Find the position vector of U in terms of **p** and **r**.
Give your answer in its simplest form.

..... [2]

(c) $\overrightarrow{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\overrightarrow{MT}| = \sqrt{180}$.

Find the positive value of k.

$k =$ ..... [3]

8 $f(x) = 2x + 1$ $g(x) = x^2 + 4$ $h(x) = 2^x$

(a) Solve the equation $f(x) = g(1)$.

$x =$ ..... [2]

(b) Find the value of $f(h(3))$.

..... [2]

(c) Find $f^{-1}(x)$.

$f^{-1}(x) =$ ..... [2]

(d) Find $g(f(x))$ in its simplest form.

..... [3]

(e) Solve the equation $h^{-1}(x) = 0.5$.

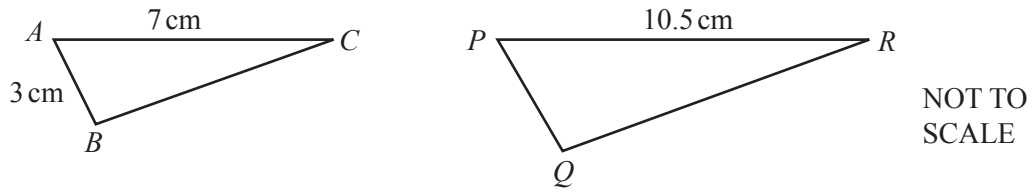
$x =$ ..... [1]

(f) $\dfrac{1}{h(x)} = 2^{kx}$

Write down the value of k.

$k =$ ..... [1]

9



The triangles ABC and PQR are similar.

(a) $AB = 3$ cm, $AC = 7$ cm and $PR = 10.5$ cm.

Calculate the length of PQ.

$PQ =$ ..... cm [2]

(b) Triangle ABC is the cross section of a prism of length 12 cm.
Triangle PQR is the cross section of a prism of length 18 cm.
The volume of the smaller prism is V cm^3.

(i) Explain why the prisms are similar.

.....

..... [1]

(ii) Find, in terms of V, the volume of the larger prism.

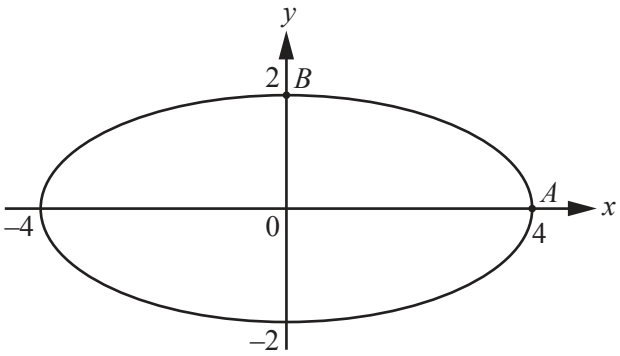
..... cm^3 [2]

(c) Angle $ABC = 110°$.

Calculate angle ACB.

Angle $ACB =$ ..... [3]

10



NOT TO SCALE

The diagram shows a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) A is the point (4, 0) and B is the point (0, 2).

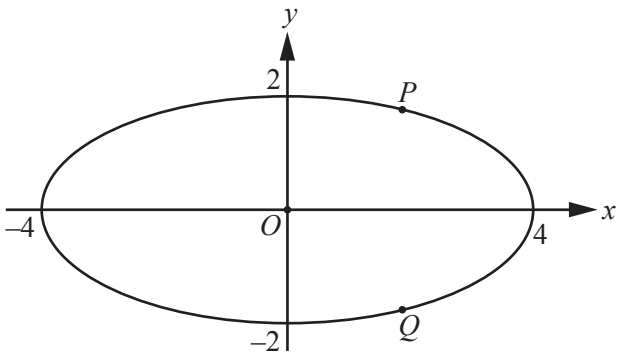
(i) Find the equation of the straight line that passes through A and B. Give your answer in the form $y = mx + c$.

$y =$ ..... [3]

(ii) Show that $a^2 = 16$ and $b^2 = 4$.

[2]

(b)



NOT TO SCALE

$P\,(2, k)$ and $Q\,(2, -k)$ are points on the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

(i) Find the value of k.

$k =$ ..... [3]

(ii) Calculate angle POQ.

Angle $POQ =$ ..... [3]

(c) The area enclosed by a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(i) Find the area enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

Give your answer as a multiple of π.

..... [1]

(ii) A curve, mathematically similar to the one in the diagrams, intersects the x-axis at $(12, 0)$ and $(-12, 0)$.

Work out the area enclosed by this curve, giving your answer as a multiple of π.

..... [2]

BLANK PAGE